International Journal of Scientific & Engineering Research Volume 4, Issue 12, December 2013
ISSN 2229-5518

The Double Weighted Rayleigh Distribution Properties and Estimation

Nasr Ibrahim Rashwan

Abstract

This paper presents a new weighted distribution which is known as the double weighted rayleigh distribution (DWRD). This distribution is constructed and studied. The statistical properties of this distribution are discussed and obtained, including the mean, variance, coefficient of variation, harmonic mean, moments, mode, coefficient of skewness, coefficient of kurtosis, reliability function, hazard function and the reverse hazard function. Also the parameters of this distribution are estimated by the method of moment and the maximum likelihood estimation method.

Keywords : *Weighted distribution, Double weighted distribution, Double weighted rayleigh distribution, Moment estimation method, Maximum likelihood estimation method.*

1. Introduction

Weighted distribution Theory gives a unified approach to dealing with model specification and data interpretation problems (biased data). Weighted distributions occur frequently in studies related to reliability, survival analysis, analysis of family data, Meta analysis, analysis of intervention data, biomedicine, ecology and several other areas. (See Stene (1981), Gupta and Keating (1985),Patil and Taillie (1989), and Oluyede and George (2002)). Many authors have presented important results on weighted distributions, Rao (1965) extended the basic ideas of the methods of ascertainment upon estimation of frequencies by Fisher (1934) and introduced a unified concept of weighted distribution and identified various sampling situations that can modeled by weighted distributions, These situations occur when the recorded observations can not be considered as a random sample from the original distributions, This mean that sometimes it is not possible to work with a truly random sample from population of interest. Zelen (1974) introduced weighted distribution to represent what he broadly perceived as a length-biased sampling. Patil and Ord (1976) studied a size biased sampling and related invariant weighted distributions. Statistical applications of weighted distributions related to human population and ecology can be found in Patil and Rao (1978). Gupta and Tripathi (1996) studied the weighted version of the bivariate logarithmic series distribution, which has applications in many fields such as: ecology, social and behavioral sciences and species abundance studies. Shaban and Boudrissa (2007) presented the weibull length biased distribution with its properties and estimated of its parameters. Jing (2010) introduced the weighted inverse weibull and a beta-inverse weibull distribution throughout studying properties. Das and Roy (2011) discussed the length biased weighted generalized rayleigh distribution with its properties. Shi et al (2012) studied the theoretical properties of weighted generalized rayleigh and related distributions. Rashwan (2013) presented the generalized gamma length biased distribution with its properties.

To introduce the concept of a weighted distribution, suppose that X is a nonnegative random variable with its probability density function (pdf) f(x), then the pdf of the weight random variable X_w is given by

$$f_W(x) = \frac{w(X)f(x)}{E(W(X))}, x \geq 0 \qquad (1)$$

Where w(x) is a nonnegative weight function and $E(w(x)) = \int_0^\infty w(x)f(x)dx$, $0 < E(w(x)) < \infty$. The random variable X_w is called the weight version of X and its distribution is related to that of X and is called the weighted distribution with weight function w(x). Note that the weight function w(x) gave a different practical examples: such as when $w(x) = x^\alpha$, $\alpha > 0$, Then the resulting distribution is called is a size biased version of X and the pdf of a size random variable X_s is defined as

$$f_S(x) = \frac{x^\alpha f(x)}{E(x^\alpha)}, x \geq 0 \text{ ، } \alpha > 0 \qquad (2)$$

Where $E(x^\alpha) = \int_0^\infty x^\alpha f(x)dx$

In equation (2) when $\alpha = 1$, then the weight function w(x) = x and the resulting distribution is called a length-biased distribution and the pdf of a length biased random variable X_L is take the following form:

$$f_L(x) = \frac{xf(x)}{E(x)}, x \geq 0 \qquad (3)$$

Where $E(x)=\mu$ is the mean of the original distribution and equal $E(x)=\int_0^\infty x f(x)dx$. (to know more details about different forms of a weight function see Rao (1985) and Hewa (2011)).

International Journal of Scientific & Engineering Research Volume 4, Issue 12, December 2013
ISSN 2229-5518

Rayleigh distribution is an important distribution in statistics and operation research. It is applied in several areas such as health, agriculture, biology and other science. Also it is considered to be a very useful lifetime distribution in the reliability theory. If a random variable X follow the rayleigh distribution with a scale parameterλ, then the probability density function is given by

$$f(x) = 2\lambda x\, e^{-\lambda x^2},\ x \ge 0,\ \lambda > 0 \qquad (4)$$

with the cumulative distribution function (pdf) is

$$f(x) = \int_o^x 2\lambda t e^{-\lambda t^2} dt = 1 - e^{-\lambda x^2}$$

$$\text{and } f(\alpha x) = 1 - e^{-\lambda \alpha^2 x^2} \quad (5)$$

This paper develops a new weighted distribution which is known the double weighted distribution especially, the double weighted rayleigh distribution (DWRD). Properties of DWRD are discussed and the parameters of this distribution are estimated and obtained by using moment method and the maximum likelihood method. A numerical example is introduced for illustration.

2. Double Weighted rayleigh distribution

The double weighted distribution is defined as follow

$$f_D(x) = \frac{w(x) f(x) F(\alpha x)}{w_D},\ x \ge 0,\ \alpha > 0 \qquad (6)$$

Where $w_D = \int_0^\infty w(x)\, f(x)\, F(\alpha x)\, dx$ and the first weight is w(x)and the second weight is(αx),F(αx) depend on the original distribution f(x), $f(x) = 2\lambda x e^{-\lambda x^2}$,$F(\alpha x) = 1 - e^{-\lambda\alpha^2 x^2}$. Taking weight function as w(x) = x, Then

$$w_D = \int_0^\infty 2\lambda x^2 e^{-\lambda x^2}(1 - e^{-\lambda \alpha^2 x^2})dx$$

Let $y = \lambda x^2,\ x^2 = \frac{y}{\lambda},\ x = \sqrt{y/\lambda},\ dx = \frac{dx}{2\sqrt{\lambda y}},\ y \ge 0$

$$= \frac{1}{\sqrt{\lambda}}\Gamma(3/2)\left[1 - \left(\frac{1}{(1+\alpha^2)}\right)^{3/2}\right]$$

$$\therefore W_D = \frac{1}{2}\sqrt{\pi/\lambda}\left[1 - \left(\frac{1}{(1+\alpha^2)}\right)^{3/2}\right] \qquad (7)$$

Using the equation 4,5 and 7, and substituting in equation 6 after considering w(x)= x, we obtain the probability density function of the double weighted rayleigh distribution as follow:

$$f_D(x) = \frac{2\lambda x^2 e^{-\lambda x^2}(1 - e^{-\lambda\alpha^2 x^2})}{\frac{1}{2}\sqrt{\pi/\lambda}[1 - (\frac{1}{1+\alpha^2})^{3/2}]}$$

$$= \frac{4\lambda^{3/2} x^2 e^{-\lambda x^2}(1 - e^{-\lambda\alpha^2 x^2})}{\sqrt{\pi}[1 - (\frac{1}{1+\alpha^2})^{3/2}]}$$

let $k = \frac{1}{1+\alpha^2}$, then

$$f_D(x) = \frac{4\lambda^{3/2} x^2 e^{-\lambda x^2}(1 - e^{-\lambda\alpha^2 x^2})}{\sqrt{\pi}(1 - k^{3/2})},\ x \ge 0,\ \lambda, \alpha > 0 \qquad (8)$$

3. The Statistical Properties of DWRD

In this section, we present the statistical properties of DWRD throughout computing the mean, variance, coefficient of variation, harmonic mean, moments, mode, coefficient of skewness, coefficient of kurtosis, cumulative distribution function, reliability function, hazard function and the reverse hazard function as follow:

- **The mean of this distribution is**

$$E(x) = \mu = \int_0^\infty x f_D(x) dx = \frac{4\lambda^{3/2}}{\sqrt{\pi}(1-k^{3/2})}\int_0^\infty x^3 e^{-\lambda x^2}(1 - e^{-\lambda\alpha^2 x^2})dx$$

Let $y = \lambda x^2,\ x^2 = \sqrt{\lambda/\lambda},\ dx = \frac{dy}{2\sqrt{\lambda y}},\ y \ge 0$

$$\therefore \mu = \frac{4\lambda^{3/2}}{\sqrt{\pi}(1-k^{3/2})}\left[\frac{1}{2\lambda^2}\Gamma(2) - \frac{1}{2\lambda^2}\Gamma(2)\left(\frac{1}{1+\alpha^2}\right)^2\right]$$

$$= \frac{2(1-k^2)}{\sqrt{\lambda\pi}(1-k^{3/2})} \qquad (9)$$

- **The variance**

$$\sigma^2 = E(x^2) - E^2(x)$$

$$E(x^2) = \frac{4\lambda^{3/2}}{\sqrt{\pi}(1-k^{3/2})}\int_0^\infty x^4 e^{-\lambda x^2}(1 - e^{-\lambda\alpha^2 x^2})dx = \frac{3(1-k^{5/2})}{2\lambda(1-k^{3/2})}$$

Then $$\sigma^2 = \frac{3\pi(1-k^{5/2})(1-k^{3/2}) - 8(1-k^2)^2}{2\lambda\pi(1-k^{3/2})^2} \qquad (10)$$

International Journal of Scientific & Engineering Research Volume 4, Issue 12, December 2013
ISSN 2229-5518

- **The coefficient of variation is**

$$CV = \frac{\sigma}{\mu} = \frac{[3\pi(1-k^{5/2})(1-k^{3/2}) - 8(1-k^{2})_{2}]^{\frac{1}{2}}}{2\sqrt{\pi}(1-k^{2})} \qquad (11)$$

- **The harmonic Mean is**

$$\frac{1}{H} = E(\frac{1}{x}) = \frac{4\lambda^{3/2}}{\sqrt{\pi}(1-k^{3/2})} \int_0^\infty x e^{-\lambda x^2}(1-e^{-\lambda\alpha^2 x^2})dx$$

$$\therefore \frac{1}{H} = \frac{2\sqrt{\lambda}(1-k)}{\sqrt{\pi}(1-k^{3/2})} \qquad \therefore H = \frac{2\sqrt{\pi}(1-k^{3/2})}{2\sqrt{\lambda}(1-k)} \qquad (12)$$

- **The shape**

The shape of the pdf given in equation (8) can be clarified by studying this function defined over the interval [0, ∞] and the behavior of its derivatives as follows:The limits of the pdf of DWRD is given by

$$\lim_{x\to\infty} f_D(x) = \frac{4\lambda^{3/2}}{\sqrt{\pi}(1-k^{3/2})} \lim_{x\to\infty} x^2 e^{-\lambda x^2}(1-e^{-\lambda\alpha^2 x^2}) = 0 \qquad (13)$$

Because $\lim_{x\to\infty} e^{-\lambda x^2} = 0$ and $\lim_{x\to\infty} e^{-\lambda\alpha^2 x^2} = 1$

From this limit we conclude the pdf of DWRD has one mode say x_0 as follow:

by taking the logarithm of $f_D(x)$, Then

$$\text{Ln } f_D(x) = \ln\left[\frac{4\lambda^{3/2}}{\sqrt{\pi}(1-k^{3/2})}\right] + 2\ln x - \lambda x^2 + \ln(1-e^{-\lambda\alpha^2 x^2})$$

Differentiating $\ln f_D(x)$ with respect to x, we obtain

$$f'(x) = f_D(x)\left[\frac{2}{x} - 2\lambda x + \frac{2\lambda\alpha^2 x \; e^{-\lambda\alpha^2 x^2}}{1-\lambda\alpha^2 x^2}\right] \qquad (14)$$

and equating this derivative to zero gives:

$$\frac{2}{x} - 2\lambda x + \frac{2\lambda\alpha^2 x \; e^{-\lambda\alpha^2 x^2}}{1-\lambda\alpha^2 x^2} = 0 \qquad (15)$$

by solving the nonlinear equation in (15) with respect x we obtain the mode.

The following table gives the mode value of DWRD with different values of the parameters of λ and α

λ	α	Mode
1	**2**	**1.02998**
1	**6**	**1.00002**
2	**3**	**0.70711**
2	**10**	**0.69795**
4	**6**	**0.49988**
8	**10**	**0.32299**
12	**12**	**0.28865**

and the following Table shows the mean, variance, coefficient of variation (CV) and the mode with different parameters values of λ and α

λ	α	Mean	Variance	CV	Mode
1	2	1.8995	0.20189	0.377597	1.02998
	3	1.15387	0.21268	0.399695	1.00049
	6	1.13287	0.22311	0.4169456	1.00002
	10	1.12961	0.22532	0.420219	1
2	2	0.84142	0.10094	0.37759	0.71194
	3	0.81590	0.106335	0.39967	0.70711
	6	0.80106	0.11156	0.416955	0.69965
	10	0.79879	0.11266	0.420196	0.69795
4	2	0.59498	0.05047	0.37758	0.50034
	3	0.57694	0.053169	0.39967	0.500001
	6	0.56644	0.055778	0.41694	0.49988
	10	0.56483	0.056389	0.42042	0.48799
8	2	0.42071	0.025236	0.37759	0.35419
	3	0.40795	0.026584	0.39968	0.353539
	6	0.40053	0.027889	0.41695	0.35346
	10	0.39939	0.028164	0.42019	0.32299

From the above table note that:

- Value of the mean and the mode decreases at fixed λ and increasingα.
- Value of the variance and the coefficient of variation (CV) increases at fixed λ and increasingα.
- Value of the mean is more than value of the mode (mean > mode), this refers to the distribution is skewed to the right.

- **The r^{th} moment is given by**

$$E(x^r) = \frac{4\lambda^{3/2}}{\sqrt{\pi}(1-k^{3/2})} \int_0^\infty x^{r+2} e^{-\lambda x^2}(1-e^{-\lambda\alpha^2 x^2})dx$$

Let $y = \lambda x^2$، $x^2 = \sqrt{y/\lambda}$ ، $dx = \frac{dy}{2\sqrt{\lambda y}}$ ، $y \ge 0$

$$E(x^r) = \frac{4\lambda^{3/2}}{\sqrt{\pi}(1-k^{3/2})}\left[\frac{1}{2\lambda^{\frac{r+3}{2}}}\Gamma(\frac{r+3}{2}) - \frac{1}{2\lambda^{\frac{r+3}{2}}}\Gamma(\frac{r+3}{2})(\frac{1}{1+\alpha^2})^{\frac{r+3}{2}}\right]$$

$$= \frac{2\Gamma(\frac{r+3}{2})(1-k^{\frac{r+3}{2}})}{\lambda^{\frac{r}{2}}\sqrt{\pi}(1-k^{3/2})} \qquad (16)$$

International Journal of Scientific & Engineering Research Volume 4, Issue 12, December 2013
ISSN 2229-5518

For the case r = 1 ، 2 ، 3 and 4 we have

$$-E(x)=\mu_1'=\frac{2(1-k^2)}{\sqrt{\lambda\pi}(1-k^{3/2})} \qquad -E(x^2)=\mu_2'=\frac{3(1-k^{5/2})}{2\lambda(1-k^{3/2})} \qquad (17)$$

$$-E(x^3)=\mu_3'=\frac{4(1-k^3)}{\lambda\sqrt{\lambda\pi}(1-k^{3/2})} \qquad -E(x^4)=\mu_4'=\frac{15(1-k^{7/2})}{4\lambda^2(1-k^{3/2})}$$

The first four central moments are

- $\mu_1 = E(x-\mu) = 0$
- $\mu_2 = E(x-\mu)^2$

$$\sigma^2=\frac{3\pi(1-k^{5/2})(1-k^{3/2})-8(1-k^2)^2}{2\pi\lambda(1-k^{3/2})^2}$$

- $\mu_3 = E(x-\mu)^3$ (18)

$$=\frac{2[4\pi(1-k^3)(1-k^{3/2})^2-9\pi(1-k^2)(1-k^{5/2})(1-k^{7/2})+16(1-k^2)^3]}{2\pi\lambda\sqrt{\lambda\pi}(1-k^{3/2})^3}$$

- $\mu_4 = E(x-\mu)^4$

$$\frac{15\pi^2(1-k^{7/2})(1-k^{3/2})^3-128\pi(1-k^3)(1-k^2)(1-k^{3/2})^2+144\pi(1-k^2)^2(1-k^{3/2})(1-k^{5/2})-192(1-k^2)^4}{4\lambda^2\,\pi(1-k^{3/2})^4}$$

- Coefficient of Skewness (SK).

Skewness is a measure of whether the distribution under study is symmetric or asymmetric. Asymmetric mean that, the distribution is skewed to the right (positively skewed) or skewed to the left (negatively skewed), this mean that the sign of the coefficient indicates the direction of the skew. The formula for SK is given

$$SK=\frac{\mu_3}{\sigma^3}$$

Where μ_3 is the third central moment and σ is the standard deviation of DWRD. Then

Then $$SK=\frac{2\sqrt{2}[4\pi(1-k^3)(1-k^{3/2})^2-9\pi(1-k^2)(1-k^{5/2})(1-k^{3/2})+16(1-k^2)^3]}{[3\pi(1-k^{5/2})(1-k^{3/2})-8(1-k^2)^2]^{3/2}} \qquad (19)$$

Note that the SK does not depend on λ but only depend on α.

- Coefficient of Kurtosis (ku)

Kurtosis is a measure of whether the distribution is flatness or peakedness relative to a normal distribution. The ku may be equal to zero, positive and negative. A zero value indicates the possibility of a mesokurtic distribution (that is normal high), a positive value indicates the possibility of aleptokurtic distribution (that is too tall) and a negative value indicates the possibility of aplatykurtic distribution (that is too flat).

The ku is defined by $ku=\frac{\mu_4}{\sigma^4}-3$

The coefficient of kurtosis for the DWRD is given by:

$$ku=\frac{15\pi^2(1-k^{7/2})(1-k^{3/2})^3-128\pi(1-k^3)(1-k^2)(1-k^{3/2})^2+144\pi(1-k^2)(1-k^{3/2})(1-k^{5/2})-192(1-k^2)^4}{[3\pi(1-k^{5/2})(1-k^{3/2})-8(1-k^2)^2]^2}-3 \qquad (20)$$

Note that, also the ku does not depend on λ but only depend on α.

The following Table shows value of the SK and Ku at different values of the parameter α because SK and Ku does not depend on λ but only depend on α

λ	SK	Ku
2	0.60	0.16
3	0.57	0.10
6	0.522	0.054
10	0.52	0.042
100	0.50	0.040

From the above table note that

- The value of SK decreases when α increases but still more than zero. This means that, this distribution is positively skewed (skewed to the right).
- The value of Ku decreases when α increases but still more than zero. This means that, this distribution is a leptokurtic distribution (more peaked than normal curve distribution).

- **The cumulative distribution function (cdf) is**

$$F_D(x)=\frac{4\lambda^{3/2}}{\sqrt{\pi}(1-k^{3/2})}\int_0^x t^2 e^{-\lambda t^2}(1-e^{-\lambda\alpha^2 t^2})dt$$

$$=\frac{4\lambda^{3/2}}{\sqrt{\pi}(1-k^{3/2})}\left[\frac{1}{2\lambda^{3/2}}\Gamma_I(3/2,\lambda x^2)-\frac{k^{3/2}}{2\lambda^{3/2}}\Gamma_I(3/2,\lambda x^2(1+\alpha^2))\right]$$

$$\therefore F_D(x)=\frac{2[\Gamma_I(3/2,\lambda x^2)-k^{3/2}\Gamma_I(3/2,\lambda x^2(1+\alpha^2))]}{\sqrt{\pi}(1-k^{3/2})} \qquad (21)$$

Where Γ_I is a generalized incomplete gamma function.

- **The reliability function or survival function R(x).**

This function can be derived using the cumulative distribution function and given by

$R(x) = 1 - F_D(x)$

$$=\frac{\sqrt{\pi}(1-k^{3/2})-2[\Gamma_I(3/2,\lambda x^2)-k^{3/2}\Gamma_I(3/2,\lambda x^2(1+\alpha^2))]}{\sqrt{\pi}(1-k^{3/2})} \qquad (22)$$

International Journal of Scientific & Engineering Research Volume 4, Issue 12, December 2013
ISSN 2229-5518

- **The hazard or instantaneous rate function H(x).**
  The hazard function of x can be interpreted as instantaneous rate or the conditional probability density of failure at time x, given that the unit has survived until x. the hazard function is defined to be

$$H(x)=\frac{f_D(x)}{R(x)}$$
$$=\frac{4\lambda^{3/2}x^2e^{-\lambda x^2}(1-e^{-\lambda\alpha^2x^2})}{\sqrt{\pi}(1-k^{3/2})-2[\Gamma_I(3/2,\lambda x^2)-k^{3/2}\Gamma_I(3/2,\lambda x^2(1+\alpha^2)]}\quad(23)$$

- **The reverse hazard function (ϕ (x))**
  The reverse hazard function can be interpreted as an approximate probability of failure in [x ، x + dx], given that the failure had occurred in [0, x]. The reverse hazard function ϕ (x) is defined to be (Finkelstein (2002)).

$$\phi(x)=\frac{f_D(x)}{F_D(x)}=\frac{4\lambda\alpha^{3/2}x^2e^{-\lambda x^2}(1-e^{-\lambda\alpha^2x^2})}{2[\Gamma_I(3/2,\lambda x^2)-k^{3/2}\Gamma_I(3/2,\lambda x^2(1+\alpha^2)]}\quad(24)$$

3. Estimation of the Parameters

In this section, estimates of the two parameters (λ,α) of the double weighted rayleigh distribution are estimated and obtained. Method of moment (MOM) and maximum likelihood estimators (MLE) are presented.

3-1. Method of moment estimators

Let x_1، x_2، … ،x_n be an independent random sample from the DWRD with parameters λand α. The method of moment estimators are obtained by computing the population moments by using

$$E(x^r)=\left[2\Gamma(\frac{r+3}{2})(1-k^{\frac{r+3}{2}})/\lambda^{r/2}\sqrt{\pi}(1-k^{3/2}),r=1,2\right]$$

and equating to the sample moments $M_r=[\frac{1}{n}\sum_{i=1}^{n}X_i^r,r=1,2]$.

The following equations are obtained using the first and second sample moments.

$$\frac{2(1-k^2)}{\sqrt{\pi}(1-k^{3/2})}=\frac{1}{n}\sum_{i=1}^{n}X_i=\overline{X}\quad(25)$$

and

$$\frac{3(1-k^{5/2})}{2\lambda(1-k^{3/2})}=\frac{1}{n}\sum_{i=1}^{n}X_i^2\quad(26)$$

Solving the two equations (25) and (26) simultaneously (numerical method), we will get $\hat{\lambda}_m$ and $\hat{\alpha}_m$ as estimate of λ and α respectively. These estimates are generally used as initial values for the maximum likelihood method when no closed form exists for the MLE and the normal equations needs to be solved iteratively.

Note that, from equation (25) when α is known we obtain estimate for λ, That is

$$\hat{\lambda}_m=\frac{4(1-k^2)^2}{\overline{x}^2\pi(1-k^{3/2})^2}\quad(27)$$

- From equation (25) when λ is known, the estimate of α, can be obtained by numerical methods.

3-2. Maximum Likelihood estimators

Let x_1، x_2، … ،x_n be an independent random sample from the DWRD, then the likelihood function of DWRD is given by

$$f_D(x_1,x_2,...,x_n;\lambda,\alpha)=\prod_{i=1}^{n}\left[\frac{4\lambda^{3/2}x^2e^{-\lambda x^2}(1-e^{-\lambda\alpha^2x^2})}{\sqrt{\pi}(1-k^{3/2})}\right]\text{، } k=\frac{1}{1+\alpha^2}\quad(28)$$

Using the above equation, the log-likelihood function (Ln) is obtained

$$Ln(\lambda,\alpha)=nLn4+\frac{3}{2}n\ln\lambda-\frac{n}{2}\ln\pi-n\ln(1-k^{3/2})+2\sum_{i=1}^{n}x_i-\lambda\sum_{i=1}^{n}x_i^2+\sum_{i=1}^{n}\ln(1-e^{-\lambda\alpha^2x^2})\quad(29)$$

By taking derivatives of the Ln (λ, α) with respect to the parameters λ and α, we obtain the following equations:

$$\frac{\partial\ln(\lambda,\alpha)}{\partial\lambda}=\frac{3n}{2\lambda}-\sum_{i=1}^{n}x_i^2+\sum_{i=1}^{n}\frac{\alpha^2x^2e^{-\lambda\alpha^2x^2}}{1-e^{-\lambda\alpha^2x^2}}\quad(30)$$

and

$$\frac{\partial\ln(\lambda,\alpha)}{\partial\alpha}=\frac{3\alpha}{(1+\alpha^2)[(1+\alpha^2)^{3/2}-1]}+\sum_{i=1}^{n}\frac{2\lambda\alpha x^2e^{-\lambda\alpha^2x^2}}{1-e^{-\lambda\alpha^2x^2}}\quad(31)$$

Equating these equations to zero, then we get the normal equations as follow

$$\frac{3n}{2\lambda}-\sum_{i=1}^{n}X_i^2+\sum_{i=1}^{n}\frac{\alpha^2x^2e^{-\lambda\alpha^2x^2}}{1-e^{-\lambda\alpha^2x^2}}=0\quad(32)$$

and

$$\frac{3\alpha}{(1+\alpha^2)[(1+\alpha^2)^{3/2}-1]}+\sum_{i=1}^{n}\frac{2\lambda\alpha x^2e^{-\lambda\alpha^2}x^2}{1-e^{-\lambda\alpha^2}x^2}\quad(33)$$

International Journal of Scientific & Engineering Research Volume 4, Issue 12, December 2013
ISSN 2229-5518

This nonlinear equations system must be solved for λ and αsimultaneously since a closed form solution is not known an iterative technique is required to compute the estimators $\hat{\lambda}_L$and $\hat{\alpha}_L$ The system of equations is solved by netwon-raphson iteration method. (See Jeffery (1992), vinod and Gaurav (2010)).

4. Numerical Example

In this section, we analyze a data set from Gupta and Akman (1995) to illustrate the estimation methods of the parameters that used in this paper (Shaban and Boudrissa (2007)). These data are:

17.88	**28.92**	**33**	**41.52**	**42.12**	**45.6**	**48.48**	**51.84**	**51.96**
54.12	**55.56**	**67.8**	**68.64**	**68.64**	**68.86**	**84.12**	**93.12**	**98.64**
105.12	**105.84**	**127.92**	**128.04**	**173.40**				

These data refer to millions of revaluations to failure for 23 ball bearings in fatigue test. Analysis of these data involved two steps:

The first a descriptive summary of a sample data is computed and presented in the following table:

Measure	Mean	Median	Mode	Variance	SK	Ku
Value	72.2243	67.8	68.64	1405.402	1.008	0.926

This summary shows a positively skewed and a leptokurtic distribution. The second, the parameters of the DWRD were estimated numerically since there was no closed form for them. By using the moment estimates method, the system non linear equations in (25) and (26) was solved numerically and yields estimators y parameters λ and α as follows: $\hat{\lambda}_m = 3.806\ x10^{-4}$ and $\hat{\alpha}_m$ = 0.123. these values used as initial values for the normal equations in (32) and (33) to obtain the maximum likelihood estimates, then $\hat{\lambda}_L = 0.005299$ and $\hat{\alpha}_L = 1.540409 \times 10^{-8}$ for 18 iterated.

5. Conclusion

This paper developed a new weighted distribution which is known as the double weighted Rayleigh distribution. Some statistical properties of this distribution are discussed and studies. It also estimates the parameters of this distribution using the method of moment and the maximum likelihood estimates method. The calculations are illustrated with the help of numerical example.

References

[1] Das, K.K. and Ray, T.D. (2011): on some Length-biased weighted weibull distribution", Pelagia Research Library, Advances in Applied Science Research, Vol.2, 465-475.

[2] Finkelstein, M.S., (2002). "on the reverse hazard rate", Reliability Engineering and System Safety, Vol.78, 71-75.

[3] Fisher, R.A., (1934), "The effects of methods of ascertainment upon the estimation of frequencies", The Annals of Eugenics, Vol.6, 13-25.

[4] Gupta, A.K., and Nadarajah, S. (1985), "relation for reliability measures under length biased sampling", Scandinavian Journal of Statistics, Vol.13,49-56.

[5] Gupta, A.K., and Tripathi, R.C., (1996), "Weighted bivariate logarithmic series distributions: commun. Statist.Theory Meth. Vol.25, 1099-1117.

[6] Hewa, A.P., (2011), "Statistical properties of weighted generalized gamma distribution", Master Dissertation, Statesboro, Georgia.

[7] Jeffery, D.H., (1992), "Maximum Liklihood estimation of dietary intake distributions", This paper prepared for the human nutrition information service of USDA, IowaStateUniversity.

[8] Jing, X.K., (2010), "Weighted Inverse weibull and Beta-inverse weibull distributions", Master Dissertation, Statesboro, Georgia.

[9] Rashwan, N.I., (2013), "A Length-biased version of the generalized Gamma distribution", Advances and Applications in Statistics, Vol.32, 119-137.

[10] Oluyede, B.O., and George, E.O., (2002) "on stochastic inequalities and comparisons of Reliability measures for weighted distributions", Mathematical problems in Engineering, Vol.8, 1-13.

[11] Patil, G.P., and Ord, J.K., (1976), "On size biased sampling and related form invariant weighted distribution", The Indian Journal of Statistics, Vol.39, 48-61.

[12] Patil, G.P., and Rao, G.R., (1978), "Weighted distributions and size biased sampling with applications to wildlife populations and human families", Biometrics, Vol.34, 179-189.

[13]Patil, G.P., and Taillie, C., (1989), "Probing encountered data, meta analysis, and weighted distribution methods in statistical data analysis and inference", Y. Dodge, ed, Elsevier, Amsterdam.

[14] Rao, C.R., (1965), "on discrete distributions arising out of methods of a ascertainment in classical and contagious discrete distributions", G.P. Patil, ed., pergamon press and statistical publishing society, Calcutta.

[15]Rao, C.R., (1985), "Weighted distributions arising out of methods of ascertainment in a celebration of statistics, A.C. Atkinson and S.E. Fienberg, eds., Springer-Verlag, New York.

[16] Shaban, S.A., and Boudrissa, N.A., (2007), "The weibull Length biased distribution-properties and estimation http/interstat.statjournals.net/index.php.

[17] Shi, X., oluyeede, B.O., and Pararai, M., (2012), "Theoretical Properties of weighted generalized Rayleigh and related distributions", Theoretical Mathematics and Applications, Vol.2, 45-62.

[18] Stene, J. (1981), "Probability distributions arising from the ascertainment and the analysis of data on human familes and other groups", statistical distributions in scientific work, applications in physical, Social and life Sciences, Vol.6, 233-244.

[19] Vinod, K., and Gaurav, S., (2010), "Maximum Likelihood estimation in generalized gamma type model", Journal of reliability and statistical studies,Vol.3, 43-51.

[20] Zelen, M., (1974), "Problems in cell kinetics and the early detection of disease, in reliability and biometry, F. Proschan and R.J. Sering, eds, SIAM, Philadelphia